Effects of Probe Geometry on Turbulence Measurements in Liquids Using Hot-Film Anemometry

J. M. Rodriguez,* G. K. Patterson,† and J. L. Zakin‡
University of Missouri—Rolla, Rolla, Mo.

Comparisons of the performances of several commonly used hot-film anemometer probes were made by measuring turbulence intensities and energy spectra at the center of a 1-in. i.d. tube for several fluids. The results indicate that differences as large as 20% may be obtained for turbulence intensities measured in purely viscous fluids, depending on probe geometry. Normalized energy spectra measured with the different types of probes were similar, in spite of the different measured values of turbulence intensities. Measurements in a viscoelastic polymer solution indicate the necessity of performing studies of the response of hot-film probes in this type of fluid.

Nomenclature

A = constant in Eq. (2)
B = coefficient in Eq. (2)
c = exponent in Eq. (4)
C_p = specific heat
d = probe cylinder diameter
D = tube inside diameter
e' = root-mean-square fluctuating voltage
E = probe voltage
$\bar{E}$ = time average probe voltage
$E(n)$ = component of the energy spectrum
$F(n)$ = component of the normalized energy spectrum in terms of frequency
$F(k)$ = component of the normalized energy spectrum in terms of wave number
I = probe current
k = wave number or thermal conductivity
l = probe length
n = frequency
N_{Nu} = Nusselt number, hd/k
N_{Pr} = Prandtl number, $C_p\mu/k$
N_{Re_c} = cylinder Reynolds number, $d\bar{U}/\nu$
N_{Re} = pipe Reynolds number, $D\bar{U}_b/\nu$
R = resistance
u = instantaneous fluctuating velocity
u' = rms velocity
U = instantaneous velocity in longitudinal direction
$\bar{U}$ = time average velocity
$\bar{U}_b$ = bulk mean velocity in pipe
μ = viscosity
ν = kinematic viscosity
ρ = density
σ = standard deviation

Introduction

THE hot-sensor anemometer with either the constant temperature or constant current mode of operation is one of the most useful research tools available to study the structure of turbulent flows. Measurements in gases using hot-wire anemometers have been performed for many years, mainly using constant current operation. The response of hot-wires has been extensively studied[1–3] and the reliability of hot-wire anemometry in gases is well established.

The use of anemometry in liquid measurements has gained prominence only in the last few years. Ling[4] introduced the hot-film probe which is more rugged and less likely to be fouled by lint and dirt than the hot-wire probe. Developments in electronics that have permitted design of more stable feed-back systems for constant temperature operation have made liquid turbulence measurements with hot-film constant temperature anemometry almost as common as gas measurements.

However, for constant temperature operation, the problems encountered with hot-film sensors are much greater than those for hot wires. The major problem is the unknown response of the hot-film sensors at high frequencies. The large mass on which the film is deposited makes these probes react more sluggishly to high-frequency velocity fluctuations. Also, the boundary layer developed around the probe may affect the response of heat-transfer rate to velocity change, since the boundary layer has a response time[5,6] that is directly proportional to the boundary-layer thickness for a given fluid at a given velocity. For gases the boundary-layer response is fast enough under most conditions, due to the very thin boundary layers, to have little effect on the over-all probe response, but for liquids the greatly increased thickness of the boundary layer causes its response to high-frequency perturbations to be often slower than the electronic response of the anemometer; thus the boundary-layer response and the sluggish film probe response are the predominant effects. Accurate measurements of the over-all frequency response of film probes in liquids have not yet been accomplished.[7,8]

Another problem involves the conductivity of some fluids, such as water. Passing an electrical current through a wire or film in an ionized liquid causes conduction in the liquid and an extraneous current at a given probe potential. This problem can be avoided by applying a thin protective coating of quartz to the film, thus avoiding direct contact of the film with the liquid. The addition of the quartz film decreases the high-frequency response of the film probe.[9]

Geometric factors also have to be considered, i.e., the effect of the probe shape, its size, and the size of the supports on the turbulence measurements. In this paper, comparisons are made of the results obtained with five commonly used sensors —two cylinders (0.006 in. and 0.001 in. diam), a cone, a wedge, and a parabolic probe—for three organic solvents and a polymer solution. Intensities of turbulence and energy spectra are compared. The energy spectra provide rough comparisons of the relative frequency responses of the different probes while the intensities of turbulence show how the flow field around a probe affects the level of the measured turbulence. The differences between measurements with these

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* Graduate Student, Department of Chemical Engineering.

† Assistant Professor, Department of Chemical Engineering.

‡ Professor, Department of Chemical Engineering.

probes indicate to what extent comparisons of results obtained by different investigators with different types of probes are valid.

Theory

Hot-film or hot-wire anemometer operation is based on the assumption that the heat generated in the film or wire is dissipated by convection to the surrounding fluid. Under constant temperature operation the resistance of the film (and thus its temperature) is fixed. The film or wire is one of the four legs of an impedance bridge. When the probe is exposed to a flowfield with a fluctuating velocity component, the fluctuations affect the rate of heat transfer from the probe. The anemometer circuit will tend to keep the temperature of the probe constant, thus changing the amount of current through the probe. Since power = energy/time = I^2R = heat transferred/time, current (and thus voltage) fluctuations are an indication of the velocity fluctuations of the flowfield.

Empirical relationships have been obtained for heat transfer by forced convection from a cylinder to a gas or a liquid. The relationship given by Kramers[10]

$$N_{\mathrm{Nu}} = 0.42\, N_{\mathrm{Pr}}{}^{0.20} + 0.57\, N_{\mathrm{Pr}}{}^{0.33} N_{\mathrm{Re}_c}{}^{0.50} \tag{1}$$

where N_{Nu} = Nusselt number = hd/k, N_{Pr} = Prandtl number = $C p\mu/k$, and N_{Re_c} = cylinder Reynolds number = $dU\rho/\mu$, correlates heat-transfer data for many gases and liquids. U is the velocity in the direction of mean flow.

For a cylinder heated by an electric current, the heat transferred per unit time from the cylinder to the fluid is given by E^2/R. Equation (1) can be written as

$$E^2 = A + BU^{0.50} \tag{2}$$

where A and B are constants for a given fluid, a given probe, and a given value of the temperature difference between the probe and the fluid. This applies to constant temperature operation where the resistance of the probe is constant. For turbulence let $E = \bar{E} + e$ and $U = \bar{U} + u$. For low values of turbulence intensity ($u'/\bar{U} \ll 1$), Eq. (2) can be written as

$$\bar{E}^2 = A + B\bar{U}^{0.50} \tag{3}$$

Although this relationship was developed for a cylinder, it has been used successfully to represent power-velocity relationships for other geometries as well. It is used for probe calibration with A and B determined experimentally. For this work, a modified form of Eq. (3) was used

$$\bar{E}^2 = A + B\bar{U}^c \tag{4}$$

where the exponent c was chosen to minimize the mean square deviations of a least-squares fit of $\bar{E}^2$ vs. $\bar{U}^c$.

To obtain intensities of turbulence, it can be shown that for low intensities (both normal and transverse to the sensor) the approximate relationship

$$\text{Intensity of turbulence} = \frac{u'}{\bar{U}} = \frac{e'}{\bar{U}(d\bar{E}/d\bar{U})} \tag{5}$$

can be used[12] where u' and e' are root-mean-square values. For intensities less than 20%, the error in $u'/\bar{U}$ is always less than 5%[12] for calibration curves of the form of Eq. (4).

Using Eq. (4), Eq. (5) can be written as

$$u'/\bar{U} = 2\bar{E}e'/cB\bar{U}^c \tag{6}$$

It should be noted that linearization circuits are available, giving a linearized relationship between $\bar{E}$ and $\bar{U}$, relaxing somewhat the restriction of low intensities normal to the sensor. The assumption of a low transverse intensity is still necessary. The results presented in this paper were obtained without a linearizer, and Eq. (6) was used for calculations. Turbulence intensities were always less than 6%.

Table 1 Critical velocity for a Reynolds number of 70 for a 0.006-in. cylinder

Fluid	Velocity, fps	Viscosity, cp at 25°C
Mineral oil	4.0	4.20
Toluene	0.48	0.560
Cyclohexane	0.82	0.889
Polymer solution	5.4	4.9[a]

[a] Low shear rate.

The one-dimensional energy spectrum is an indication of the kinetic energy associated with velocity fluctuations of different frequencies in a given direction. $E(n)$, the energy associated with a frequency n, is defined such that

$$\int_0^\infty E(n)dn = u'^2 \tag{7}$$

or for the case of energy over discrete frequency intervals

$$\sum_{i=1}^{\infty} E(n_i)(\Delta n)_i = u'^2 \tag{8}$$

A normalized energy spectrum is defined such that

$$\int_0^\infty F(n)dn = 1 \tag{9}$$

$F(n)$ and $E(n)$ are related by

$$F(n) = E(n)/u'^2 \tag{10}$$

Even though a three-dimensional energy spectrum can be defined[11] only the one-dimensional spectrum is of concern in this paper.

Experimental

Intensity and energy spectrum measurements were made in a 1-in. i.d. tube. The flow development length was approximately 300 diam. Measurements were taken at the tube axis and at several radial positions up to a value of r/R = 0.70. The fluids used are listed with their viscosities in Table 1. The probes were centered in a short section of a dummy tube with the same inside diameter as the test section. The traversing mechanism with the probe thus centered was then removed from the dummy tube and attached to the test section tube. Symmetry checks about the pipe axis indicated that the procedure was accurate to ± 0.01 in., or an r/R of ± 0.02. Since only centerline measurements were compared, this amount of uncertainty in position is not significant. The temperature of the fluid was kept at 25°C ± 0.05°C. Overheat ratios of 1.08-1.10 were used, thus giving probe temperatures in the range of 53 to 70°C. Five types of probes were used—a wedge probe, a 0.006-in. and a 0.001-in.-diam cylinder probe, a cone probe, and a parabolic probe. All but the wedge probe were quartz coated. The specifications of these probes are given in Table 2. The cylinder

Table 2 Characteristics of sensing surface

Type of probe	Materials of construction	Sensors dimensions
Cylinder	Quartz-coated platinum	0.006 in. diam × 0.080 in. long
Cylinder	Quartz-coated platinum	0.001 in. diam × 0.020 in. long
Wedge	Platinum	0.2 mm wide × 1.0 mm long
Cone	Quartz-coated platinum	0.008-in. band 0.010 in. in diameter (0.03 in. long) behind a 30° conical tip
Parabolic	Quartz-coated platinum	about 0.002 in. wide × 0.030 in. long

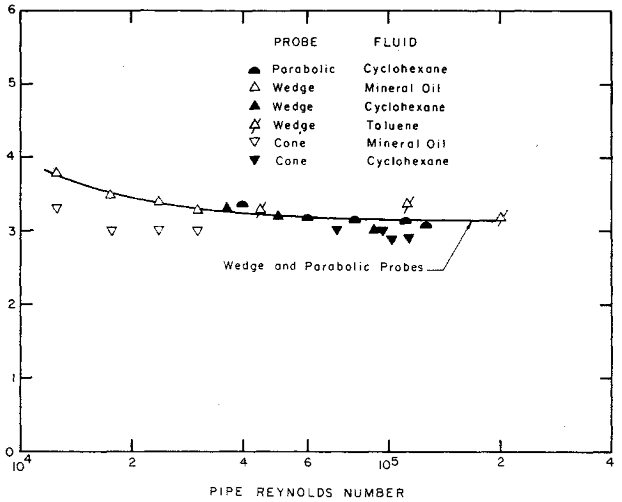

Fig. 1 Turbulence intensity data for wedge, cone, and parabolic film probes in solvents.

probes had low length to diameter ratios when compared to the usual wire probe. Cones, wedges, and parabolic wedges by the nature of their shape do not collect dirt and lint as readily as cylinders and thus are easier to use in liquids.

A problem encountered with the use of cylinder probes is eddy shedding behind the cylinder. There are three flow regimes around a cylinder.[13] Below a cylinder Reynolds number of 0.5 the flow is completely laminar. Above a Reynolds number of 10^5 the flow is completely turbulent. Between 0.5 and 10^5, the flow is unstable and a wake forms behind the cylinder. At Reynolds numbers in the range of 50–100, there is a noticeable increase in drag, which corresponds to the region where eddy shedding commences. Table 1 shows the velocity corresponding to a Reynolds number of 70 for the different fluids studied in this investigation for a 0.006-in. cylinder. Reynolds numbers were calculated using estimated viscosities at the temperature of the probe surface. It is clear that for the range of velocities studied (5–25 fps), the flow around the 0.006-in. cylinder probe is not laminar and eddy shedding will occur.

Fabula[6] studied the response of a 0.006-in. cylinder probe using water as the test fluid. He observed eddy shedding to occur, and this appeared as a ripple in the output from the constant temperature anemometer. The frequency of the ripple increased as velocity (or Reynolds number) increased. At a velocity of 3 fps, he observed a ripple frequency of about 800 cps. To avoid this extraneous signal in the 0.006-in. cylinder probe intensity measurements reported here, all contributions above 1000 cps were filtered. A ripple frequency of 1000 cps would correspond to a velocity of about 4 fps for cyclohexane, 3 fps for toluene.[14] The lowest velocity used in the work reported here was 8 fps. At low flow rates, the error introduced by neglecting the part of the signal above 1000 cps is negligible, as can be seen from an energy spectrum. At high flow rates, the error introduced is not large, but is significant. Wells, Harkness, and Meyer[15] indicate that for pipe Reynolds numbers in the range of those used in this work (up to 10^5), about 90% of the energy is below 1000 cps. Thus, the error introduced will be small and will always tend to give low intensities with the cylinder probe, if the low-frequency response is correct.

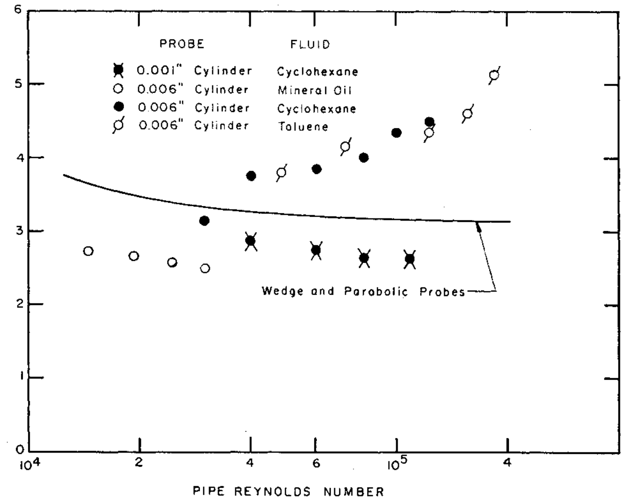

Fig. 2 Turbulence intensity data for cylinder film probes in solvents.

Energy spectra were obtained by performing a Fourier analysis of tape recordings of the output of the constant temperature anemometer with a set of 34 band-pass filters that covers the range 10–20,000 cps.[16] The band pass filters each cover $\frac{1}{3}$ octave and their center frequencies are arranged in accordance with ISO standards. The tape recorder has a flat frequency response in the range 60–20,000 cps using direct recording channels and in the range 5–100 cps using pulse duration modulation (PDM) channels, both at a recording speed of 6 in./sec. Using a direct and a PDM channel it is possible to cover a wide range of frequencies with uniform response. Recordings taken with cylinder probes were not filtered at 1000 cps.

Discussion of Results

Turbulence Intensities

Intensities of turbulence were measured for four fluids: cyclohexane, toluene, mineral oil, and a 0.40% solution of polyisobutylene (PIB L-200, Enjay Chemical Company) in cyclohexane. Measurements in toluene were performed with only the wedge and 0.006-in. cylinder probes. The 0.001-in. cylinder probe and the parabolic probe were used only with cyclohexane. Figures 1–3 show the results reported as intensity at the center of the tube in percent of local time average velocity vs Reynolds number based on tube diameter. Intensities were calculated as discussed in the experimental section using Eq. (6). Table 3 gives the values of *A* and *B* used to calculate the data. It should be noticed that most

Table 3 Least-squares fit of calibration curves

Fluid	Probe	*A*	*B*	*c*	σ on $\bar{E}^2$, %
Cyclohexane	0.006-in. cylinder	98.1	76.90	0.50	1.0
Cyclohexane	0.001-in. cylinder	6.88	16.12	0.50	0.7
Cyclohexane	Wedge	43.64	117.01	0.50	0.8
Cyclohexane	Cone	97.35	28.18	0.50	0.7
Cyclohexane	Parabolic	30.43	22.15	0.50	0.8
Toluene	0.006-in. cylinder	7.97	34.99	0.45	1.0
Toluene	Wedge	19.77	64.83	0.45	0.6
Mineral oil	0.006-in. cylinder	5.29	24.00	0.50	1.5
Mineral oil	Wedge	13.86	37.06	0.50	0.6
Mineral oil	Cone	17.10	16.50	0.40	1.9
Polymer solution	0.006-in. cylinder	53.51	121.35	0.40	0.9
Polymer solution	Wedge	34.29	36.41	0.45	2.5
Polymer solution	Cone	14.68	15.69	0.35	0.6

of the data fit an exponent C of 0.50, which corresponds to Eq. (3). The precision of fit for each set of data is indicated by the standard deviation of the data from calculated values. Worst standard deviation is 2.5% but most are less than 1.0%.

The wedge and cone probes gave very repeatable turbulence intensity results. Several replications of the runs with a wedge probe gave results that are generally within ±5% of each other. The cylinder probe data for 0.40% PIB L-200 in cyclohexane were difficult to obtain, because of erratic changes in calibration. We believe that the cylinder data for the polymer solution are good to ±10%. The mineral oil, toluene, and cyclohexane cylinder data are more reliable since several replications of the experiment gave results that were within a ±5% confidence range of the average value.

Figure 1 shows intensity data obtained with the wedge probe for the three solvents. The data are consistent and gave a single line independent of the fluid. Also shown are the data obtained with a parabolic probe in cyclohexane. The wedge and parabolic probes behaved identically as should be expected, since their geometries are very similar.

Figure 1 also presents results for the cone probe in mineral oil and cyclohexane. The cone probe results with mineral oil and cyclohexane are 5–15% lower than the wedge probe with the smaller deviation at the highest Reynolds number. These deviations are within the range of experimental error in hot-film anemometry but the fact that the cone data are lower than the wedge in both cases indicates that there is a constant bias.

Figure 2 presents cylinder probe data as a function of tube Reynolds number, which is proportional to cylinder Reynolds number for a given cylinder size. The 0.001-in. diam cylinder data parallel the wedge probe data, but are about 25% lower. The 0.006-in. cylinder probe shows a great dependence on tube (and therefore cylinder) Reynolds number. Intensities measured at low Reynolds numbers are lower than the wedge results, but as the Reynolds number increases, so do the values of turbulence intensities. For toluene, the least viscous fluid, intensities as high as 5% were measured with the cylinder compared to 3% with the wedge. These measurements are independent of solvent as can be seen by comparison of the toluene and cyclohexane data in the same Reynolds number range. It must be remembered that the turbulence signals for the 0.006-in. cylinder data were filtered at 1000 cps, and the measured intensities should be low. The fact they are high indicates that the flowfield is disturbed by the probe in such a way as to affect the velocity fluctuation measurements below 1000 cps.

These high intensities are related to the cylinder Reynolds number. The 0.001-in. cylinder probe does not show this effect in the range of flow rates attainable in these measurements. The increase in intensity for the 0.006-in. cylinder probe starts to appear at a tube Reynolds number of 25,000 which corresponds to cylinder Reynolds number of 400, taking into account the overheat of the fluid film around the cylinder. For the smaller cylinder probe, this would not happen below a tube Reynolds number of 150,000 in a 1-in. tube. Below this Reynolds number the data should not be anomalous. As shown in Fig. 2, intensities measured with a cylinder at low cylinder Reynolds numbers are lower than those measured with a wedge, a cone, or a parabolic probe.

The polymer solution data are shown in Fig. 3. The wedge probe gives results similar to those obtained for pure solvents, but this is not always true for polymer solutions.[17,18] The cone and cylinder probes give results which differ markedly from wedge probe results. The cone probe data are higher than the wedge at low Reynolds numbers but cross the wedge data at a pipe Reynolds number of 15,000. A completely different result was obtained with the 0.006-in. cylinder probe. Intensity results were higher than those obtained with the wedge probe. This is the opposite of what was observed for mineral oil. However, Virk[19] has reported measurements using a 0.002-in. cylinder probe in a 0.1% polyethylene oxide-water solution, and his measured value at the center of the tube was 4%. This is close to the values obtained here at higher Reynolds numbers.

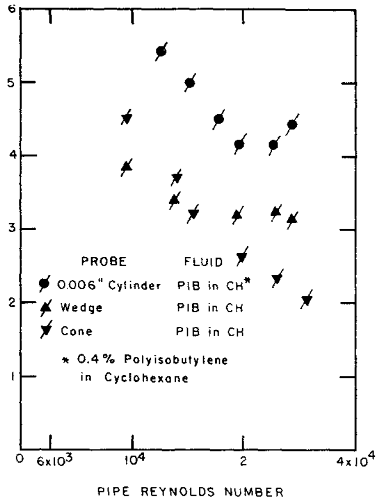

Fig. 3 Turbulence intensity data for wedge, cone, and cylinder film probes in a 0.4% PIB in cyclohexane solution.

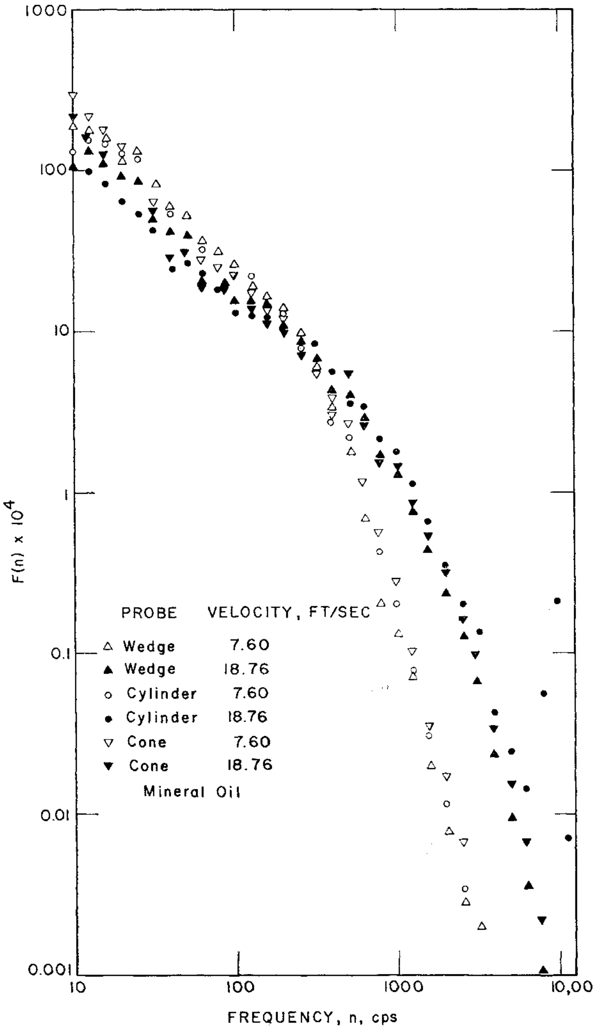

Fig. 4 Turbulence energy spectra for wedge, cone, and cylinder film probes in mineral oil at two velocities; plotted as function of frequency.

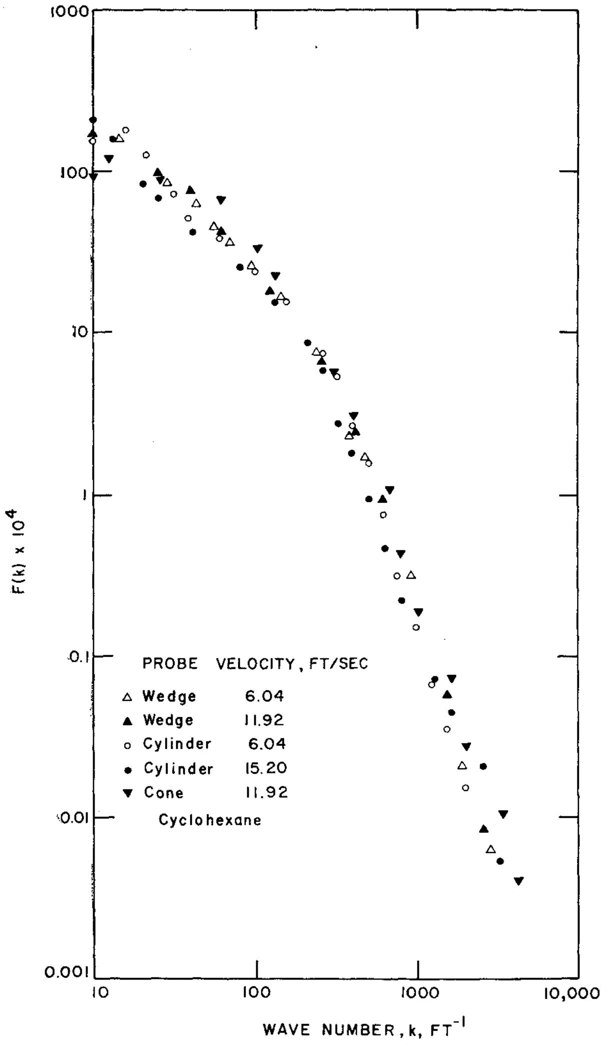

Fig. 5 Turbulence energy spectra for wedge, cone, and cylinder film probes in cyclohexane; plotted as function of wave number.

These complicated effects indicate that the analysis of the response of hot-film anemometer probes to velocity fluctuations in viscoelastic fluids by Metzner and Astarita[20] was not complete. Their analysis shows that hot-film probes should yield low turbulence intensity results, but evidently both abnormally high and low turbulence intensities may be observed depending on probe geometry and Reynolds number. It is clear that it is necessary to study the response of hot-film probes to known velocity fluctuations in viscoelastic fluids in order to resolve these discrepancies. The major objective of this paper is to show the pure solvent measurements, the polymer measurements being presented only to indicate the state-of-the-art in this type of system.

Energy Spectra

Energy spectra were measured in mineral oil, cyclohexane, and the polymer solution using the wedge, cone, and 0.006-in. cylinder probes. For comparison purposes normalized spectra will be considered. Not all spectra were measured at the same flow rate, so comparisons will be made using normalized spectra in terms of wave number $k = 2\pi n/\bar{U}_b$, which leads to a definition of $F(k)$ given by

$$F(k) = \bar{U}_b F(n)/2\pi$$

Normalized wave number spectra are shown in Fig. 5 to collapse the spectrum data for a given fluid at different flow rates into a single curve in the high wave number region.

Figure 4 shows the energy spectra as a function of frequency for mineral oil at Reynolds numbers of 11,500 and 29,000. The significant observation is that the spectra measured with the three probes are nearly identical at low and high flow rates except for a small spread at low frequencies. No drop in the high-frequency response of the cylinder probe is observed in this case, which conflicts with the observations of Fabula.[6] A large peak at 9500 cps was observed with the cylinder probe at the highest flow rate. This is due to eddy shedding.

Figure 5 shows energy spectra as a function of wave number for cyclohexane measured with the cone, the wedge, and the 0.006-in. cylinder probes. Spectra measured in this solvent with the three types of probes agree very well at high frequencies.

It should be noted that the highest wave numbers for these energy spectra correspond to lengths much shorter than the film lengths on the probes. Since the wedge and cone film lengths were about the same, any corrections to the high wave number ends of the spectra would be about the same. It is doubtful that the effect was serious, however, because the cylinder film was twice as long as the wedge film, yet measured nearly identical spectra.

Figure 6 presents energy spectrum data for the polymer solution. For each probe the wave number plot reduces the data for various velocities to a single curve for both high and low wave numbers. Intensities measured with the 0.006-in. cylinder probe were about 1.5 times greater than solvent intensities obtained with the same probe in the same Reynolds number range. The wave number spectra show that the cylinder data are generally lower than the cone and wedge data at high wave numbers. The wedge and cone probes indicate similar normalized energy spectra, even though the level of turbulence intensities measured by the wedge and the cone are different.

It should be remembered that most of the turbulence energy is in the low frequency region. If there is a change in the local structure of turbulence about the cylinder or low-frequency eddy shedding that will give high intensities but at the same time not affect the high frequencies, this would appear in a normalized spectrum as an increase of the contribution of the low frequencies and a decrease of the high frequencies.

There is some evidence that the high intensities observed in polymer solutions with the cylinder probes was caused by higher turbulence energy levels at low frequency. Comparing the spectra of Fig. 6 at $k = 600$ ft^{-1}, a value of 3×10^{-5} is obtained for the line through the data for the cone and wedge probes and a value of 7×10^{-6} is obtained for the cylinder probe data. The ratio of these two values indicates that the total kinetic energy measured (which is mainly at low wave numbers) in the cylinder measurement would have to be increased by a factor of about 4 (the intensity by 2) to account for the observed shift in the normalized spectra at high frequencies. Since that factor is very close to the factor for the increase of turbulence intensity measured with the cylinder probe over that measured with the wedge and cone probes (about 1.5), the cylinder probe seems to have lost no high-frequency response, but merely indicated anomalously high low-frequency response. Since the Reynolds numbers for these measurements are in the same range as the oil measurements, where the loss in high-frequency response was not observed, the change in frequency-response characteristics of the cylinder probe can be explained only in terms of the properties of the polymer solution. Viscoelasticity has an effect on the frequency-response characteristics of cylinder probes that should be investigated in detail.

Conclusions

1) Turbulence intensity measurements with a wedge film probe in toluene, cyclohexane, and mineral oil are in close

agreement with cyclohexane data obtained with a parabolic film probe. Cone film probe data in mineral oil and cyclohexane are consistently 5–15% lower than the wedge and parabolic probe data.

2) Intensity measurements with cylinder probes in the same three solvents are dependent on the cylinder Reynolds number. At low Reynolds numbers they are lower than the wedge probe, but above a cylinder Reynolds number of 400 they increase and are higher at high Reynolds numbers even when the eddy shedding energy is filtered out. It is believed that this effect is due to wake disturbances caused by the size of the cylinder and its support. The use of small diameter cylinders would avoid this anomalous behavior.

3) Intensity measurements in a viscoelastic polymer solution gave values both higher and lower than solvents at the same Reynolds number depending on the probe geometry and the flow rates. Only the wedge probe gave results close to those of the Newtonian solvents with the polymer solution investigated.

4) Normalized energy spectra for mineral oil were virtually the same when measured by a wedge, a cone, and a 0.006-in. cylinder probe. Data for different velocities and the three different probes fit a single curve when plotted as normalized wave number spectra despite widely different intensity levels. Similar results were obtained for cyclohexane. This result conflicts with earlier comparative data on similar cylinders.[6]

5) Normalized wave number energy spectra for a polymer solution at different velocities were identical when measured with wedge and cone film probes. The high-frequency region of the spectra for the higher intensity 0.006-in. cylinder probe measurements was lower. This is probably due to anomalously high response for this probe at low frequencies in the viscoelastic solutions as compared with the wedge and cone probes.

6) A direct study of frequency-response characteristics for some type of hot-film probe will be necessary in order to determine the error involved in energy spectrum measurements. Spectrum errors involved in the use of other probes may be determined by comparisons of absolute energy spectra.

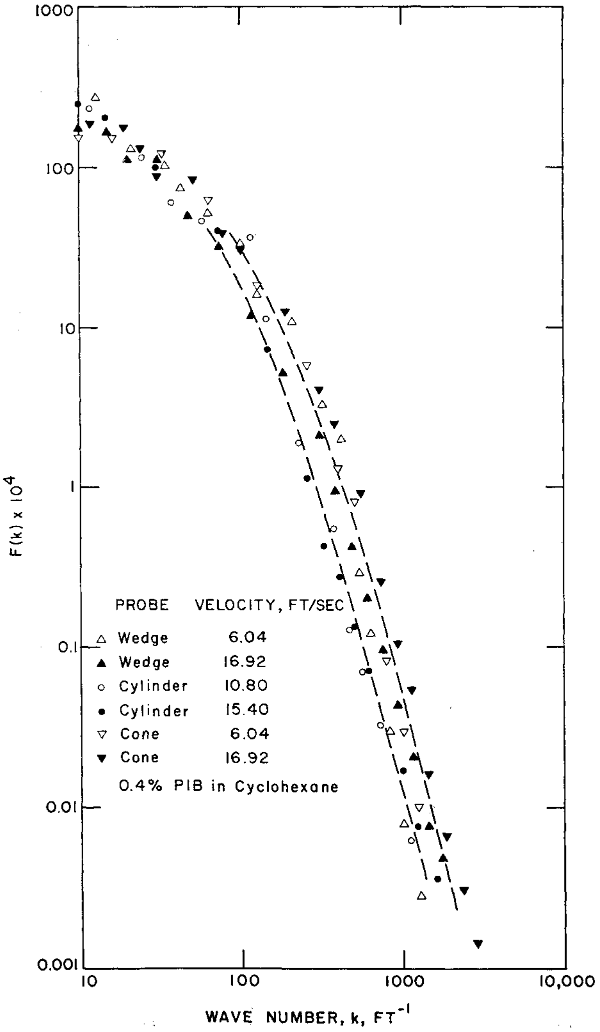

Fig. 6 Turbulence energy spectra for wedge, cone, and cylinder film probes in a 0.4% PIB in cyclohexane solution.

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